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The Calculation of Frequency Source Requirements for Digital Communications Systems

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Abstract—**Frequency sources, such as oscillators and synthesizers, are critical to the proper operation of digital communications systems. Frequency sources are used as local oscillators (LO's) in RF systems, data clocks in all systems, and sampling clocks in digitally implemented systems. Frequency sources affect critical communications system parameters such as acquisition, loss of lock, and bit error rate (BER) performance. The issues for the first two items are well known to the time and frequency community--those of synchronization, syntonization, and cycle slipping. This paper focuses on item three, how frequency source parameters affect BER performance. The paper is tutorial in nature, rather than an exhaustive treatment, and is meant for those familiar with time and frequency theory but not digital communications theory. A quadrature phase shift keyed (QPSK) RF communication system is for illustrative examples, but the methods outlined in this paper are generally applicable to all types of digital communications systems.**

Keywords-frequency sources; requirements; digital; communications systems.

I. INTRODUCTION

Frequency sources, such as oscillators and synthesizers, are critical to the proper operation of digital communications systems. Frequency sources are used as local oscillators (LO's) in RF systems, data clocks in all systems, and sampling clocks in digitally implemented systems. Frequency sources effect critical communications system parameters such as acquisition, loss of lock, and bit error rate (BER) performance. How frequency sources affect the first two items, acquisition and loss of lock, involve issues of synchronization, syntonization, and cycle slipping, and the theory is well known to those in the time and frequency community. The third item, how frequency sources affect BER performance, is less well known to those in the time and frequency community and requires an understanding of the nature of the processes that cause BER degradation in digital communications systems. This paper will concentrate on this third item. The paper is tutorial in nature, rather than an exhaustive treatment, and is meant for those familiar with time and frequency theory but not digital communications theory. A quadrature phase shift keyed (QPSK) RF communication system is used for illustrative examples, but the methods outlined in this paper are generally applicable to all types of digital communications systems.

II. COMMUNICATIONS SYSTEMS CONCEPTS

A. Signals Carrying Digital Information

Figure 1 shows some of the basic terms and concepts used in digital communications. The reader is referred to the general references [1,2,3,4] for further information on these basic concepts. A digital communications system modulates a carrier signal with a temporal series of symbol waveforms representing the digital information being transmitted. The carrier signal, for example, may be an RF tone or just a DC voltage. Shown in the figure are unshaped symbols modulated on a DC voltage carrier called pulse amplitude modulation (PAM). Unshaped symbols consist of rectangular modulation waveforms of clock period T_c centered about a series of decision epochs $t_n = nT_c$ such that the symbol waveforms do not overlap from clock period to clock period. There are also other types of symbol waveforms called shaped symbols that do overlap in time. These will be discussed later.

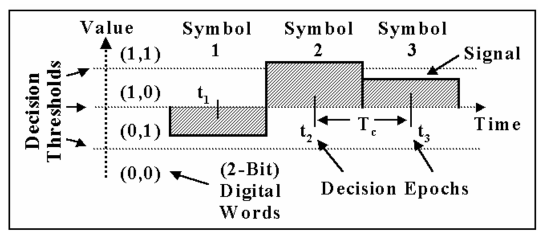

Figure 1. Basic terms and concepts (Example: unshaped PAM symbols).

At the decision epoch t_n, the sampled symbol waveform has a modulation value V_n. This value, when decoded, represents a digital word S_n, which contains W digital bits of information. To determine S_n from V_n, the value axis or space is separated into regions by decision thresholds. In the simplest form of decoding, the word S_n is assigned to the value V_n when V_n falls into the decision region associated with that word. In more complex decoding schemes, such as that used for error correction [5,6], a collection of N V_n values is used in aggregate to determine an associated collection of N words S_n. In the PAM example shown in Figure 1, the decision thresholds divide the value axis into four regions, each representing a 2-bit word.

The symbol rate R_s at which symbols are transmitted in symbols per second is given by

$$R_s = 1/T_c. \quad (1)$$

The data rate R in bits per second is given by

$$R = W\,R_s = W/T_c. \quad (2)$$

B. Shaped Symbols

As shown in Figure 2a, an unshaped symbol is rectangular in the time domain, extending $\pm T_c/2$ from the decision epoch such that each symbol doesn't interfere with the next. In the frequency domain, as shown in Figure 2b, this produces a sync function spectrum that extends well beyond a band of $\pm R_s/2$. If one limits the bandwidth of this spectrum (with an analog filter) to something close R_s, the rectangular pulse is distorted such that one symbol overlaps another. This leads to inter-symbol interference (ISI), which causes degradation in the performance of the digital system.

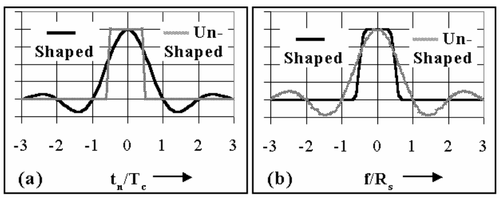

Figure 2. (a) Symbols in the time domain. (b) Symbols in the frequency domain.

Shaped symbols [1] resolve this problem by being more bandwidth efficient in the frequency domain without introducing ISI. Figure 2a also shows a shaped symbol in the time domain. A shaped waveform is a sinc-like function that extends beyond $\pm T_c/2$, but has zeros at symbol epochs other than its own, so there is no ISI. The spectrum of a shaped symbol is shown in Figure 2b. Note that it has a rectangular-like frequency spectrum, extending to a little over $\pm R_s/2$. Thus shaped symbols are inherently bandwidth efficient without additional filtering. Figure 3 shows a digital transmission consisting of multiple shaped symbols. Note that only one symbol contributes a value at each decision epoch. The price one pays for shaping is that ISI increases faster with data clock error for shaped symbols than that for unshaped ones.

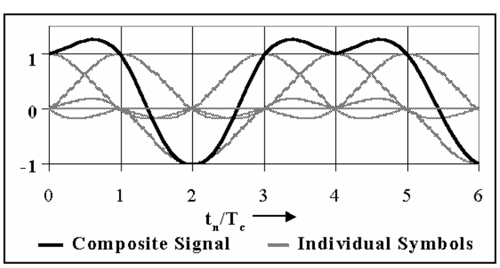

Figure 3. Digital transmission consisting of multiple shaped symbols.

C. Inter-Symbol Interference (ISI) and Eye Patterns.

ISI is the contamination of the received value V_n representing symbol S_n at the decision epoch t_n by other symbols. An eye pattern, as shown in Figure 4 [7], is very useful in understanding the nature of ISI and its dependence on decision epoch timing. An eye pattern consists of the value trajectories plotted as a function of time modulo 1-symbol period (as in a scope trace synchronized to the symbol clock). Each open eye in the pattern is a region with no trajectories for a certain data word state (2^W states for W bits per symbol). The width of the trajectories at a given decision epoch for adjacent eye edges shows the ISI for that data value. For an ideal system, there is no ISI at the ideal decision epoch, and the width of such trajectories goes to zero at that epoch as shown in the figure. In non-ideal systems, either the ISI never goes to zero due to channel distortion and/or timing jitter makes the decision epoch wander around so the ISI doesn't appear to go to zero. The ratio of the height of the eye opening to its width yields a sensitivity coefficient for ISI verses timing error. Shaped symbols generate narrower eye widths than unshaped ones and are thus more sensitive to timing error.

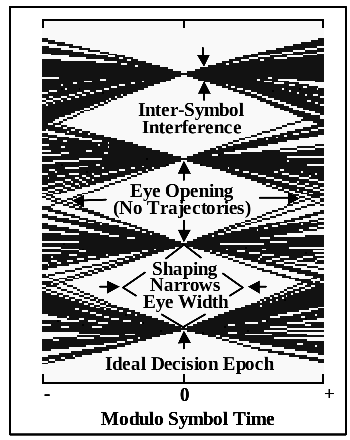

Figure 4. Eye pattern [7].

D. Types of Digital Modulation.

Digital modulation can be categorized in several ways as follows.

Type of Carrier. The carrier can be an RF carrier or subcarrier, a DC voltage, an optical signal, etc. Some parameter of the carrier is modulated with symbols as described next.

Parameter Modulated. Various carrier parameters can be used for modulation. RF carriers can be phase shift modulated or keyed (PSK), frequency shift keyed (FSK), pulse amplitude modulated or amplitude shift keyed (PAM or ASK), or pulse width or position modulated (PWM or PPM). DC carriers can be PAM, PWM, or PPM. There are hybrids such as quadrature amplitude modulation or shift keyed (QAM or QASK), which consists of amplitude modulation of the in-phase and quadrature components of an RF carrier, and coherent phase-frequency shift keying, which keeps the phase coherent while shifting the frequency (CPFSK).

Modulation Order. The modulation order is the number of digital states per symbol or 2^W. Thus binary refers to a 1-bit symbol, quadrature refers to 2-bit symbol, 8 refers to a 3-bit symbol, etc. M-ary is a reference to a general order, where $\log_2(M)$ is the number of bits per symbol.

Shaped or Unshaped Symbols. This has been described previously.

Phase coherence (of RF carriers). Phase coherence refers to the necessary coherence properties of the phase in RF carrier systems. There are three categories: coherent, incoherent, and differential. The definition and impact of these categories on RF carrier phase jitter requirements will be described later.

Synchronous or Asynchronous Data Clock Timing. Synchronous data clock timing refers to the fact that the data clock synchronization is supplied along with the symbol stream as a separate line at the digital receiver. In asynchronous data clock timing, the receiver figures out the synchronization from a set preamble of symbols sent by the transmitter or other means. The terms synchronous and asynchronous are generally used in hardline transmission systems. The use of a clock recovery loop at a data receiver in RF transmission makes the system technically asynchronous, though the term is not used in this case.

E. Bit Error Rate (BER) and BER Degradation.

The bit error rate (BER) is the probability that a received bit is incorrect and the symbol error rate is the probability that one or more bits in a symbol are incorrect. The BER is a function of the signal-to-noise ratio (SNR) at the digital receiver. For an ideal system, thermal noise at the receiver is bandlimited at the symbol rate R_s. Thus the SNR at an ideal receiver for a received power P_{rx}, the power per bit P_b, and energy per bit E_b are given by

$$\text{SNR (Ideal)} = P_{rx}/(N_oR_s) = P_b/(N_oR) = E_b/N_o, \tag{3}$$

where

$$P_b = P_{rx}/W, \tag{4}$$

$$E_b = P_b/R = P_bT_c, \tag{5}$$

and N_o is the noise density per Hertz.

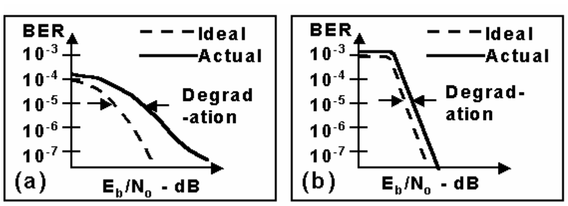

Figure 5. Typical BER vs E_b/N_o curves: (a) Uncoded. (b) Coded.

Figure 5 shows typical BER vs E_b/N_o curves, both for error correction coded (ECC) and uncoded systems. As discussed earlier, an error correction code [5,6] corrects up to N-bit errors in a group or block of bits. This is achieved by sending extra parity bits along with the data bits. One talks of a rate P/Q (i. e., 2/3) code where P/Q is the ratio of data bits to total bits transmitted. Thus, the price one pays for error correction is having to send a higher transmission bit rate (P/Q)R to achieve a user data bit rate of R. Note that the data rate always refers to R, the rate of user information data bits being sent, even when it is not explicitly stated, so the E_b/N_o in BER vs E_b/N_o curves for coded systems is still given by the uncoded formula (3). ECC improves the BER over an uncoded system above a certain threshold E_b/N_o. Note also that both ECC and uncoded BER vs E_b/N_o curves are steep waterfall curves even on the log-log plots shown. This is important when considering the effects of ISI on BER.

There are two curves on each graph marked ideal and actual. Each type of digital modulation system (and ECC) has a unique theoretical or ideal BER vs E_b/N_o curve associated with it, which is independent of R. This ideal curve assumes no ISI and ideal noise filtering of bandwidth R_s. Actual BER vs E_b/N_o curves have higher BER's at the same E_b/N_o because of ISI, and actual curves are dependent on R. The difference in E_b/N_o between the actual and ideal curve for the same BER is called the BER degradation and is usually given in dB. Note that BER degradation is a function of the BER level (or the E_b/N_o).

F. Jitter and BER Degradation.

Figure 6 shows why ISI causes BER degradation in QPSK systems.

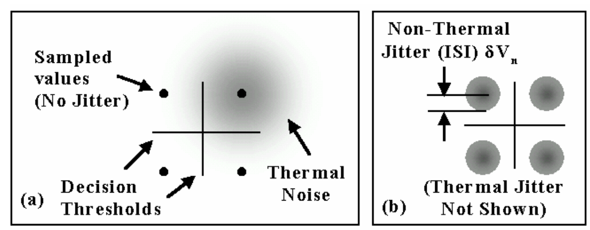

Figure 6. ISI and BER degradation: (a) Ideal system. (b) System with ISI.

In an ideal QPSK system, the sampled values without thermal noise have no jitter, and the magnitude of the sampled values $V(\pm 1 \pm j)/2^{0.5}$ of the complex RF envelope is

$$V = (P_{rx})^{1/2}. \tag{6}$$

When thermal noise is present, a gaussian jitter distribution of V_n about V is produced with a standard deviation of

$$\sigma = (N_oP_\sigma)^{1/2}. \tag{7}$$

The probability that this distribution will cross a decision threshold is given by [1]

$$\text{BER} = \tfrac{1}{2}\,\text{Erfc}(2^{-0.5}\sigma/V) = \tfrac{1}{2}\,\text{Erfc}((E_b/N_o)^{1/2}) \tag{8}$$

Note that (8) is technically the symbol error rate [1], but for small error rates this is essentially the same as the BER.

In an actual system (without thermal noise), as shown in the second part of Figure 6, the ISI causes non-thermal jitter δV_n that sometimes moves the sampled values closer to a decision threshold and sometimes further away. When the sampled values are closer to the decision thresholds, the conditional BER is higher for the same E_b/N_o, and lower when the values are further away. Because BER vs E_b/N_o curves are highly non-linear, the conditional BER's for the values closer to the thresholds dominate, and the net effect is always to degrade the total BER.

There are three basic sources of such ISI: channel distortion, carrier phase errors (in RF carrier systems), and clock errors. Channel distortion, due to filtering and non-linearities in communication channel, distorts the symbol shape and thus generates a bounded form of ISI. Carrier phase errors (in RF transmission), and clock errors generate unbounded ISI and impact frequency source requirements.

Based on Figure 7, one can generate two simple models for approximating the effects of ISI on BER degradation. These are important for generating top-level requirements without full-blown simulations of the digital system. One is the worst-case model, which is more appropriate for bounded ISI (such as channel distortion). In this model, one calculates the worst-case ISI δV_p, and uses the formula

$$\text{BER Deg (in dB)} = -20\text{Log10}(1-\delta V_p/V_o), \qquad (9)$$

where V_o is the nominal distance between the symbol (without ISI) and the nearest threshold. This model gets its name from assuming that the worst case occurs all the time and produces a constant BER degradation verses E_b/N_o.

The other is the noise model, which uses the signal-over-noise plus interference (SNIR) instead of the SNR to calculate the BER. First the SNIR $(E_b/N_o)'$ as a function of the E_b/N_o from the SNR in (3) and an σ_V^2 the standard variance of the ISI δV_n is calculated using

$$(E_b/N_o)' = ((E_b/N_o)^{-1} + (\sigma_V^2/P_{rx}))^{-1} \qquad (10)$$

Then the theoretical BER vs E_b/N_o with $(E_b/N_o)'$ substituted for E_b/N_o is used to calculate the BER degradation. This model is more universally used in calculating the effects of ISI for the purposes of flowing down specs and produces a BER degradation that increases with E_b/N_o called tailing. However, this model doesn't yield the right BER verses E_b/N_o shape for bounded ISI distributions, where the worst-case model is more appropriate.

III. Phase Jitter Requirements for RF Carrier Systems

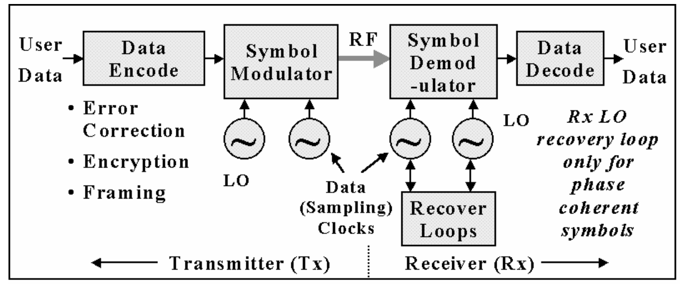

Figure 7. RF carrier digital communications system .

Figure 7 shows a block diagram of an RF carrier digital communications system. The system consists of a transmitter (Tx), an RF transmission link, and a receiver (Rx). In the transmitter, user data first enters a data encoder that adds any error correction, encryption, and framing (added structure used for Rx-Tx data synchronization) to the user data. The encoded data is then sent to a symbol modulator, which outputs an RF carrier (or subcarrier) modulated by the symbols. A data clock provides the timing for the symbols, and one or more local oscillators (LOs) generate the RF carrier. After transmission of through the RF link, a symbol demodulator in the Rx produces encoded digital words from the input RF signal and a data decoder extracts the output user data.

In all RF digital communications systems, a clock recovery loop is necessary to synchronize the Rx clock to the Tx clock (unless an external synchronization mechanism is used such as GPS). If phase coherent demodulation is used (to be defined later), a carrier recovery loop is necessary to phase track the Rx LO to the Tx LO. It is important to note that the recovery loops track out the relative Tx-Rx clock and LO jitter for fourier frequencies below the recovery loop bandwidths. This is very important for defining the appropriate jitter statistics in terms of power spectral densities (PSDs) as will be discussed later.

A. *Categories of Phase Coherence.*

How and whether the LO phase jitter effects system BER performance is determined by the phase coherence properties symbol modulation scheme used. There are three categories of phase coherence:

Coherent Phase. In a coherent phase system, the phase of an individual Tx symbol relative to the Rx LO phase is important for decoding. The Rx LO phase must be tracked to the Tx phase through a carrier recovery loop in order to maintain coherence. The bandwidth B_p of this recovery loop must be much less than the symbol rate R_s so that thermal noise induced phase jitter in the recovery loop bandwidth does not degrade the BER performance of the system. Thus $B_p^{-1} >> R_s^{-1} = T_c$, and the Rx phase is independent of the Tx phase over many symbols. Because of this, coherent phase systems have the most severe LO phase jitter requirements.

Incoherent Phase. In an incoherent phase system, the absolute phase of the RF carrier is unimportant. Examples are PAM and incoherent FSK symbol modulation. These modulation types do not need carrier recovery phase lock loops, though one may be used in an incoherent FSK system for recovering the frequency at the Rx. Note that the intra-symbol phase is important in an incoherent FSK system because it affects the modulation frequency.

Differential Phase. In differential phase systems, the data is coded so the change in phase from symbol-to-symbol carries the digital information. To decode the symbol, the phase of each symbol is compared to that of the previous one and no carrier recovery loop is required (only a one clock period delay). Thus the phase coherence only matters from individual symbol-to-symbol. One pays a price for this, however, because the effect of thermal noise and Tx LO phase jitter is increased by $2^{1/2}$ because adjacent symbols are phase differenced. Thus, the BER verses E_b/N_o performance of differential phase systems are worse than those of coherent phase systems.

In the next section, we will discuss the derivation of LO jitter requirements for coherent phase systems. These have the most stringent LO jitter requirements and are also the most often used because they have the best BER verses E_b/N_o performance. The discussion below can be easily generalized to differential phase and incoherent FSK systems by appropriately modifying jitter statistic parameters.

B. Carrier Phase Jitter and ISI.

Figure 8 shows how RF carrier phase jitter produces ISI in QPSK phase-modulated systems. RF carrier phase jitter causes the received symbol phase to wobble relative to the phase axis set by the Rx LO. This causes the Q-channel to crosstalk into the I-channel at the receiver I-decision circuit, generating ISI, and visa versa. The RMS ISI generated is $V{*}\mathrm{Sin}(\sigma_\phi)$, where σ_ϕ is a carrier phase jitter deviation statistic to be defined later. For BPSK there is no quadrature signal, so this kind of ISI doesn't occur. For BER, the jitter has to actually degrade the power received in order for BER degradation to occur and is thus a function of $(1-\mathrm{Cos}(\sigma_\phi))$. Thus BPSK is much less sensitive to phase errors. In higher-order M-ary PSK or QAM, the details are different, but the effect is similar to that of QPSK.

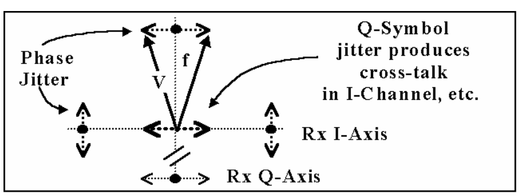

Figure 8. ISI caused by RF carrier phase jitter.

C. Calculating LO Phase Jitter for Coherent Systems.

For coherent systems, the phase jitter variance can be defined as

$$\sigma_\phi^2 = 2\int_0^{R_s/2} L_\phi(f)\,|1-H_p(f)|^2\,df \cong 2\int_{B_p}^{R_s/2} L_\phi(f)\,df \qquad (11)$$

where $L_\phi(f)$ is the sum of the single sideband PSDs of all the LOs in the system, $H_p(f)$ is the carrier recover loop response function, B_p is the loop bandwidth, and an ideal brick-wall lowpass Rx filter of bandwidth $R_s/2$ is assumed (equivalent RF bandpass of R_s). Note that the highpass response $H_p(f)$ from the recovery loop, which has an f^4 dependence below B_p, generates a finite integral in the presence of $1/f^n$ noise for $1 > n \le 4$ and the Rs/2 highpass cut-off takes care of problems for $n \ge 1$. Thus, even though (11) is the equation for a standard variance, it is finite in the presence of non-stationary LO phase jitter. This is why communications theory can treat non-stationary LO phase jitter using stationary statistics. For QPSK systems, a rule of thumb is that σ_ϕ should be 1-3 ° for BER degradations of 0.1 dB or less.

Figure 9 shows typical LO phase PSD requirements for 0.5 ° of phase jitter as a function of R_s. These curves assume that: (1) there is only white phase noise and flicker of frequency noise present, (2) both contribute equally to the phase noise, and (3) $B_p = 0.01R_s$ up to 100 KHz (and then it is fixed). The gray curve is the requirement for an LO that must operate in a system that is capable transmissions with symbol rates from 10 Hz to 1 MHz.

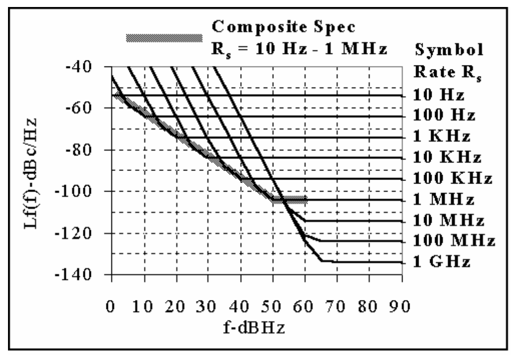

Figure 9. LO phase PSD curves for 0.5 ° of allocated phase jitter.

D. LO Requirements for Vibration Induced Phase Jitter.

Oscillators are sensitive to vibration through their g or acceleration sensitivity, usually expressed as a coefficient H_g in fractional frequency variation y (= $\delta f/f$) per g units. When exposed to a double-sided vibration power spectrum $S_g(f)$, the oscillator generates a fractional frequency double-sided spectrum $S_y(f)$ given by

$$S_y(f) = |H_g(f)|^2\,S_g(f) \qquad (12)$$

Note that the fourier frequency dependence of $H_g(f)$ should be included for exact analysis. Equation (12) can be converted to a phase noise spectrum by noting that [8]

$$S_\phi(f) = \left[\frac{f_o}{f}\right]^2 S_y(f) \qquad (13)$$

so that the phase jitter due to vibration sensitivity becomes

$$\sigma_\phi^2 = \int_0^{R_s/2} S_g(f)\left[\frac{f_o}{f}\right]^2 |H_g(f)|^2 |1 - H_p(f)|^2 \, df \quad (14a)$$

$$\sigma_\phi^2 \cong \int_{B_p}^{R_s/2} S_g(f)\left[\frac{f_o}{f}\right]^2 |H_g(f)|^2 \, df. \quad (14b)$$

Note that the high-pass cut-off from the recovery loop filtering in (14) again means the integral is finite.

From (14), one can obtain typical oscillator H_g requirements by using the typical commercial aircraft vibration spectrum shown in Figure 10 [9]. This figure shows double sideband vibration levels $S_g(f)$ with and without a vibration damper that is used to reduce high frequency vibrations.

Figure 11 shows typical H_g requirements verses symbol rate with and without a vibration damper. Since the commercial vibration levels are rather benign, the curves in Figure 11 are scaled from Figure 10 by the peak S_g level without a damper ($S_g = 0.003$ g^2/Hz is the peak value in Figure 10). The curves of Figure 11 assume that: (1) the allocated phase jitter for vibration is 0.25 º, (2) the carrier frequency is 10 GHz, (3) phase noise and flicker of frequency noise are equally contributing to the phase jitter, (4) and $B_p = 0.01R_s$. Note that the most severe H_g requirements are for the lowest symbol rates and that the vibration damper helps reduce the H_g requirements for the higher symbol rates.

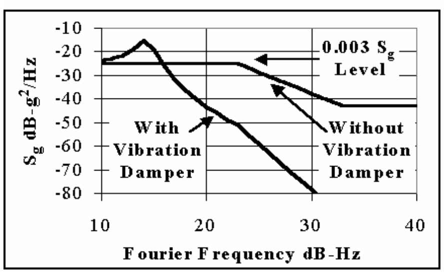

Figure 10. Typical commercial aircraft vibration levels [9].

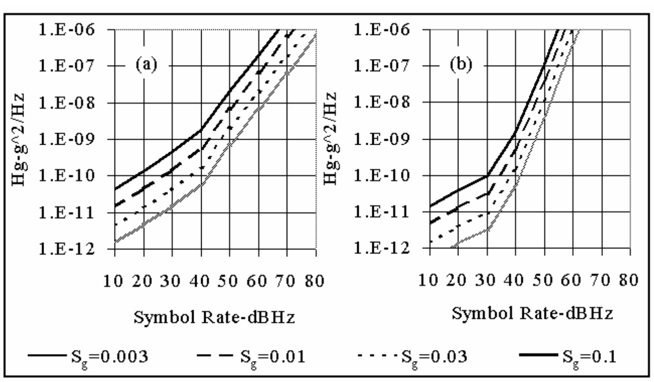

Figure 11. LO g-sensitivity requirements as a function of symbol rate for 0.25° of allocated phase jitter and 10 GHz f_o: (a) without vibration damper and (b) with vibration damper.

IV. Clock Jitter Requirements for Data and Sampling Clocks

Clock jitter from data and sampling clocks affect BER performance. Decision epoch jitter from data clocks causes BER degradation by generating ISI. Sampling or aperture clocks are used with analog-to-digital (A/D) and digital-to-analog (D/A) converters in digital implementations of transmitters and receivers. Sampling jitter reduces the effective number of bits (ENOB) of these converters causing BER degradation. The following discusses how jitter in each of these types clocks affects the BER performance.

A. Decision Epoch Jitter from Data Clocks.

The analysis of decision epoch jitter from data clocks is similar to that of phase jitter from LOs. The key to the analysis is determining a clock jitter requirement value from eye pattern and other analysis and BER degradation allocations. Once this jitter value is established, one can write the clock jitter variance in terms of the single sideband PSD of the clock reading error [8] $L_x(f)$ as

$$\sigma_x^2 = 2\int_0^{R_s/2} L_x(f)\,|1 - H_p(f)|^2 \, df \quad (15a)$$

$$\sigma_x^2 \cong 2\int_{B_p}^{R_s/2} L_x(f)df \quad (15b)$$

where x the clock reading error is given by [8]

$$x = \frac{\phi}{2\pi R_s} \quad (16)$$

Generally σ_x scales with T_c the clock period, so σ_x can be written as

$$\sigma_x = \varepsilon T_c \quad (17)$$

A rule of thumb is that ε ranges from 0.003 to 0.009 for a BER degradation from decision epoch jitter of less than 0.1 dB. Figure 12 shows typical decision epoch jitter requirements verses symbol rate using this rule of thumb.

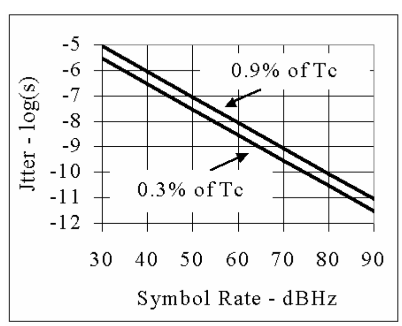

Figure 12. Typical decision epoch jitter requirements vs symbol rate.

One can also write (15) and (17) in terms of data clock phase jitter using (16). With this conversion, (15) becomes the same formula as (11), where $L_\phi(f)$ now refers to the sum of the PSD's of all the clocks in the system, and (17) is converted to

$$\sigma_\phi = 2\pi\varepsilon\,, \qquad (18)$$

where the phase jitter is in radians. The clock jitter rule of thumb translates into a data clock phase jitter of 1-3 ° for less than 0.1 dB of BER degradation. Thus Figure 9 can be used for clock PSD's verses symbol rate if we assume the same 0.5 ° jitter requirement and the same B_p rules.

B. Sampling (Aperture) Clock Jitter for A/D's and D/A's.

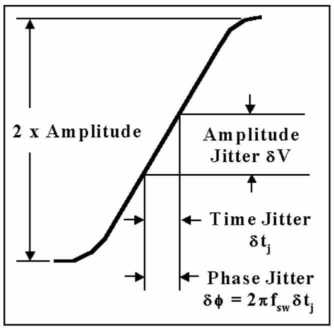

Figure 13. Impact of sampling clock jitter on digitized sine wave [10].

When a sine wave is digitized as shown in Figure 13 [10], sampling clock jitter, whose RMS value is δt_j, produces a random voltage noise whose RMS value is δV because the time jitter makes the sampled voltage randomly walk up and down the sinewave. This voltage noise produces phase jitter in the digitized sine wave whose worst-case RMS value is given by

$$\delta\phi = \delta V/A = 2\pi f_{sw}\delta t_j, \qquad (20)$$

where f_{sw} is the sinewave frequency and A is the sinewave amplitude. This limits the SNR in dB of the digitized sine wave to

$$\mathrm{SNR(dB)} = 20\mathrm{Log}(\delta\phi^{-1}) = -20\mathrm{Log}(2\pi f_{sw}\delta t_j) \qquad (21)$$

The net effect of this jitter is to limit the effective number of bits (ENOB) to [10]

$$\mathrm{ENOB} = (\mathrm{SNR(dB)}-1.76)/6.02 \qquad (22)$$

The effect of sampling clock jitter on SNR and ENOB verses sine wave frequency is shown in Figure 14 [10]. Note that because of the Nyquist criterion, the sampling clock frequency must be at least twice the sine wave frequency and is usually 3-4 times the sine wave frequency for real systems.

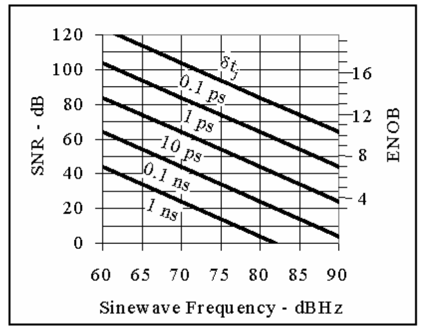

Figure 14. Effect of sampling clock jitter on A/D and D/A SNR and ENOB vs full scale sine wave frequency [10].

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